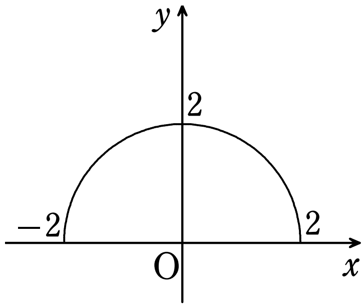
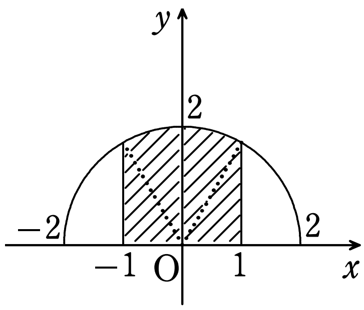
(1) $x^2+y^2=4 \iff y^2=4-x^2 \iff y=\pm\sqrt{4-x^2}$ より,

$y=\sqrt{4-x^2}$ は, 原点が中心, 半径が 2 の円の $y \geqq 0$ の部分.

よって, グラフの概形は右図となる.




(2) $\displaystyle\int_{-1}^{1}\sqrt{4-x^2}dx$ は右図の斜線部分の図形の面積に相当.




$$\therefore \int_{-1}^{1}\sqrt{4-x^2}dx = 2\times\left(\frac{1}{2}\times 2^2\times\frac{\pi}{6}+\frac{1}{2}\times 1\times\sqrt{3}\right)$$

$$= \underline{\boldsymbol{\frac{2}{3}\pi+\sqrt{3}}}$$

(2)【別解】

$y=\sqrt{4-x^2}$ は偶関数なので, $\displaystyle\int_{-1}^{1}\sqrt{4-x^2}dx = 2\int_{0}^{1}\sqrt{4-x^2}dx$

$x=2\sin\theta$ とおくと $\dfrac{dx}{d\theta}=2\cos\theta \iff dx=2\cos\theta d\theta$

| | |
|---|---|
| $x$ | $0 \to 1$ |
| $\theta$ | $0 \to \frac{\pi}{6}$ |

$$\left(\text{与式}\right)=2\int_{0}^{\frac{\pi}{6}}\sqrt{4-4\sin^2\theta}\cdot 2\cos\theta d\theta = 2\int_{0}^{\frac{\pi}{6}}\sqrt{4\cos^2\theta}\cdot 2\cos\theta d\theta$$

$$=8\int_{0}^{\frac{\pi}{6}}\cos^2\theta d\theta = 8\int_{0}^{\frac{\pi}{6}}\frac{1+\cos 2\theta}{2}d\theta = 4\left[\theta+\frac{\sin 2\theta}{2}\right]_{0}^{\frac{\pi}{6}}$$

$$=4\left(\frac{\pi}{6}+\frac{\sqrt{3}}{4}\right)= \underline{\boldsymbol{\frac{2}{3}\pi+\sqrt{3}}}$$